On the necessary conditions for the occurrence of the "environment-induced superselection rules"

Miroljub Dugić

Department of Physics, Faculty of Science

P.O.Box 60, 34 000 Kragujevac, FR Yugoslavia

E-mail address : dugic@uis0.uis.kg.ac.yu

Abstract : We briefly summarize the main recently obtained results concerning existence of the (effective) necessary conditions for the occurrence of the "environment-induced superselection rules" (decoherence).

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There is considerable interest in the subject of decoherence in the different arreas of modern physics.

Recently [1, 2], in the context of the "environment-induced superselection rules (EISR)" (or decoherence) theory [3, 4], it was pointed out existence of the (effective) necessary conditions for the occurrence of the EISR (decoherence) effect. Here we briefly summarize the main results in this concern, and distinguish some lines of the research, which is still in progress.

We shall first precisely present the methodological basis and the statements concerning the necessary conditions. Later, we briefly discuss the physical meaning of the results, with an emphasis on the question of universality of the EISR effect.

The EISR theory [3,4] relies upon the, so-called, "orthodox method", i.e. upon the formalism of the unitary operator of evolution in time, $\hat{U}(t)$, of the composite quantum system, "(open) system plus environment (S+E)". More precisely, if $\hat{\rho}_{S+E}(t=0)$ represents the initial state of the composite system, then $\hat{\rho}_{S+E}(t)$ reads :

$$\hat{\rho}_{S+E}(t) = \hat{U}(t)\hat{\rho}_{S+E}(t=0)\hat{U}^{\dagger}(t), \tag{1}$$

while the "tracing out" defines the (sub)system's "density matrix" $\hat{\rho}_S(t)$:

$$\hat{\rho}_S(t) = tr_E \hat{\rho}_{S+E}(t), \tag{2}$$

The method is the very basis of all the proper methods in the field (such as the "path integration technique", and the "master equations" formalism), and bears model-independence. This fact also follows from the analysis by Zurek [3] which distinguishes that,

effectively, all the calculations refer to the spectral form of the interaction Hamiltonian of the composite system, $\hat{H}_{int}$.

On the basis of the next set of assumptions : (i) $\hat{H}_{int}$ is time independent, (ii) $\hat{H}_{int} = \sum_{m,n} \gamma_{mn} \hat{P}_{Sm} \otimes \hat{\Pi}_{En}$ (where appear the projectors onto the subspaces of the Hilbert space of the system, $H^{(S)}$, and of environment, $H^{(E)}$) - which is [1, 2] refered to as the separability of $\hat{H}_{int}$, and (iii) Initial state of the environment is normalizable one, Zurek [3] has obtained the next results, which have been used [1, 2] as a *formal definition* of the EISR effect. The results of Zurek are as folows. In a basis $\{|m\rangle\}$ of $H^{(S)}$, which is adapted to the decomposition of $H^{(S)}$ defined by the projectors $\hat{P}_{Sm}$ in the above point (ii) - which is refered to as the "pointer basis" [3] - one obtains:

$$\rho_{Smm'} = C_m C^*_{m'} z_{mm'}(t), \tag{3}$$

where the correlation amplitude, $z_{mm'}(t)$, for each $m \neq m'$ satisfies :

$$\text{(a)} \lim_{t\to\infty} \lim_{N\to\infty} z_{mm'}(t) = 0,$$

$$\text{(b)} \lim_{t\to\infty} \langle z_{mm'}(t)\rangle_t = 0,$$

$$\text{(c)} \lim_{t\to\infty} \Delta z_{mm'}(t) \propto N^{-1/2}$$

where N-being a number proportional to the number of particles in the environment, and $\Delta z_{mm'}(t)$-being the "standard deviation" of $z_{mm'}(t)$.

In his recent paper [4], Zurek has introduced another element of definition of the EISR effect :

$$\text{(d)} \hat{U}(t)|m\rangle_S \otimes |\chi\rangle_E = |m\rangle_S \otimes |chi_m(t)\rangle_E,$$

for arbitrary initial state of the environement, $|\chi\rangle_E$.

The condition (d) is a formal expression of the requirement [4] for stability of the "preferred set of states". Since the "preferred set" needs not to consist in mutually exactly orthogonal states, the above point (d) represents a slight idealization, but which can be derived (cf. Appendix II of Ref. [2]) from the original statement.

Bearing in mind the formal definition of decoherence - i.e., the above points (a)-(d) - the occurrence of the EISR effect has been investigated [1, 2] by adopting the assumptions oposite to the above assumptions (i)-(iii). Since the assumptions underlying the defining points (a)-(d) have all been adopted as a part of the definition, the results to be presented below refer exactly, but not necessarily exclusively, to the EISR theory [3, 4].

The main result of the analysis [1, 2] is existence of the (effective) *necessary conditions* for the occurrence of the EISR effect. These conditions are as follows: (A) Diagonalizability of $\hat{H}_{int}$ in a noncorrelated basis of the Hilbert state space of the composite system - which is refered to as separability of $\hat{H}_{int}$, and (B) "Nondemolition" character of $\hat{H}_{int}$: $[\hat{H}_{int}(t), \hat{H}_{int}(t')] = 0$.

Effectiveness of the conditions refers to the cases in which the necessary conditions are not fulfilled, but for which the points (a)-(d) prove justified. In the context of the EISR theory these cases represent the pathological cases. In a (hypothetical) wider theory (which, by definition, should involve the original EISR theory) these cases appear as the particular exceptions of the next rule :

(R1) Whenever the necessary conditions are not satisfied, the EISR effect does not occur. [Note : existence of the "exceptions" has not been proved, but is just not forbidden.]

The necessary conditions have a simple physical meaning : they represent the conditions of *existence* of the "pointer basis" of the open system S. In other words: apart from the possible exceptions, if any of the above conditions ((A) or (B)) is not satisfied, the pointer basis of the open system does not exist. And this should not be missed with the cases in which pointer basis exists, but for which EISR does not occur due to some details [1] in the model ; this essentially refers to nonexistence of the sufficient conditions of EISR.

The necessary conditions directly refer to the question of universality of the EISR effect. On the other side, the question of universality seems to be fundamental in the context of the problem of "transition from quantum to classical" [5].

It seems that in the context of the EISR theory [3, 4], the universality is an implicit and plausible statement, which can be expressed by the next rule :

(R2) Appart from the "technical" details, for the realistic physical models, whenever the system is in unavoidable interaction with its environment, there occurs the EISR effect.

The rule (R2) is a statement of the original theory [3-5]. The rule (R1) is a result of the recent progress [1, 2] in the field. It is apparent that, now, the rule (R1) makes the rule (R2) essentially irrelevant, and puts a limit on the universality of the EISR effect.

This brings us to the questions and problems which follow on the basis of the rule (R1). Here we just distingusih the main operational tasks, likewise the problem of deeper physical meaning of the rule (R1).

As the main operational task in this concern appears the task of investigating separa-

bility (and/or "nondemolition" character) of $\hat{H}_{int}$. The investigation refers to the existing models, and, in addition, points to the criteria for modeling the interaction Hamiltonians in the context of the quantum measurement, and the EISR theory; for an example of the later see Ref. [6].

On the other side, the problem of deeper physical meaning and consequences of the rule (R1) calls for making the different strategies and the corresponding research programms,

and represents an investigation which is still in progress.

We conclude that existence of the (effective) necessary conditions for the occurrence of the EISR effect opens some questions in the field. In a hypothetical wider-EISR theory, the conditions lead to establishing the new rule, (R1), of the theory, and puts speciffic limit concerning universality of the EISR effect. Further researche is expected to involve both, dealing with some operational tasks (such as investigating separability of $\hat{H}_{int}$), likewise investigating the deeper physical meaning and consequences of the necessary conditions.

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